RAMAPO-INDIAN HILLS SCHOOL DISTRICT

Dear Ramapo-Indian Hills Student:

Please find attached the summer packet for your upcoming math course. The purpose of the summer packet is to provide you with an opportunity to review prerequisite skills and concepts in preparation for your next year's mathematics course. While you may find some problems in this packet to be easy, you may also find others to be more difficult; therefore, you are not necessarily expected to answer every question correctly. Rather, the expectation is for students to put forth their best effort, and work diligently through each problem.

To that end, you may wish to review notes from prior courses or on-line videos (www.KhanAcademy.com, www.glencoe.com, www.youtube.com) to refresh your memory on how to complete these problems. We recommend you circle any problems that cause you difficulty, and ask your teachers to review the respective questions when you return to school in September. Again, given that math builds on prior concepts, the purpose of this packet is to help prepare you for your upcoming math course by reviewing these prerequisite skills; therefore, the greater effort you put forth on this packet, the greater it will benefit you when you return to school.

Please bring your packet and completed work to the first day of class in September. Teachers will plan to review concepts from the summer packets in class and will also be available to answer questions during their extra help hours after school. Teachers may assess on the material in these summer packets after reviewing with the class.

If there are any questions, please do not hesitate to contact the Math Supervisors at the numbers noted below.

Enjoy your summer!

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RAMAPO INDIAN HILLS REGIONAL HIGH SCHOOL DISTRICT

AP CALCULUS AB

SUMMER PACKET

Dear Future AP Calculus AB student,

First and foremost, welcome to AP Calculus and congratulations on your enrollment, which reflects a testament to your hard work and mathematic achievements over your high school career. This letter and assignment serves to enlightenen you on the requirments and expectations for the course, and provides you with an opportunity to review and hone in on the skills necessary for success in calculus.

AP Calculus AB prepares students for the national college board examination administered in May. Therefore, enrollment in the course necessitates the following expectations for the students: they have mastered the prerequisite skills noted below, they maintain an enthusiastic and consientous attitude and work ethic for the duration of the course, they understand and can handle the pacing and work load the course requires to adequatly prepare students for the exam, and lastly, they register and sit for the AP examination in May. The main topics on the exam include differential and integral calculus; therefore, limited time in class is spent reviewing elementary functions; refreshing these skills is done by the student over the summer.

Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include those that are **linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined.** In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions of the numbers 0, pi/6, pi/4, pi/3, pi/2, and their multiples.

Course Goals

Students should be able to:

- work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems.
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- model a written description of a physical situation with a function, a differential equation, or an integral.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

As stated above, most of the year must be devoted to topics in *differential* and *integral* calculus. These topics are the focus of the AP Exam. Therefore, the summer assignments listed below are designed to help you review topics from algebra, geometry, and precalculus so that when you arrive in September, you are ready to review the first main theme in Calculus: *limits and continuity.*

- All assignments will be collected the first week back of school.
- Please complete all work in pencil and on loose leaf paper.

Assignments

I.

- ❑ Pages 8-9, #'s 1-4 all, 29, 37, 41, 45, 49, 51, 55, 57, 63, 65, 69, 83-86 all

II.

- ❑ Pages 16-17, #'s 29, 33, 39, 41, 45, 51-57 odd, 61, 63, 71

III.

- ❑ Pages 27-28, #'s 1, 5, 7, 9, 11, 15, 17, 19, 29, 33, 35, 37, 41, 43, 49-54 all, 59, 60

IV.

- ❑ Concepts worksheets 1.2-1.6

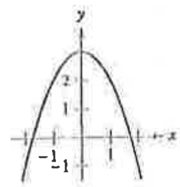
Have a great summer!

P.1 Exercises

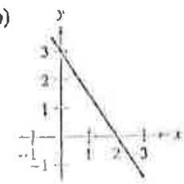
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–4, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]

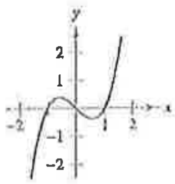
(a)



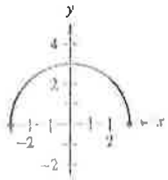
(b)



(c)



(d)



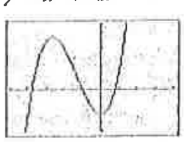
1. $y = -\frac{3}{2}x + 3$
2. $y = \sqrt{9 - x^2}$
3. $y = 3 - x^2$
4. $y = x^3 - x$

In Exercises 5–14, sketch the graph of the equation by point plotting.

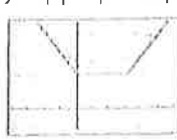
5. $y = \frac{1}{2}x + 2$
6. $y = 5 - 2x$
7. $y = 4 - x^2$
8. $y = (x - 3)^2$
9. $y = |x + 2|$
10. $y = |x| - 1$
11. $y = \sqrt{x} - 6$
12. $y = \sqrt{x + 2}$
13. $y = \frac{3}{x}$
14. $y = \frac{1}{x + 2}$

In Exercises 15 and 16, describe the viewing window that yields the figure.

15. $y = x^3 + 4x^2 - 3$



16. $y = |x| + |x - 16|$



In Exercises 17 and 18, use a graphing utility to graph the equation. Move the cursor along the curve to approximate the unknown coordinate of each solution point accurate to two decimal places.

17. $y = \sqrt{5 - x}$ (a) $(2, y)$ (b) $(x, 3)$
18. $y = x^5 - 5x$ (a) $(-0.5, y)$ (b) $(x, -4)$

In Exercises 19–28, find any intercepts.

19. $y = 2x - 5$
20. $y = 4x^2 + 3$
21. $y = x^2 + x - 2$
22. $y^2 = x^3 - 4x$
23. $y = x\sqrt{16 - x^2}$
24. $y = (x - 1)\sqrt{x^2 + 1}$
25. $y = \frac{2 - \sqrt{x}}{5x}$
26. $y = \frac{x^2 + 3x}{(3x + 1)^2}$
27. $x^2y - x^2 + 4y = 0$
28. $y = 2x - \sqrt{x^2 + 1}$

In Exercises 29–40, test for symmetry with respect to each axis and to the origin.

29. $y = x^2 - 6$
30. $y = x^2 - x$
31. $y^2 = x^3 - 8x$
32. $y = x^3 + x$
33. $xy = 4$
34. $xy^2 = -10$
35. $y = 4 - \sqrt{x + 3}$
36. $xy - \sqrt{4 - x^2} = 0$
37. $y = \frac{x}{x^2 + 1}$
38. $y = \frac{x^2}{x^2 + 1}$
39. $y = |x^3 + x|$
40. $|y| - x = 3$

In Exercises 41–58, sketch the graph of the equation. Identify any intercepts and test for symmetry.

41. $y = 2 - 3x$
42. $y = -\frac{3}{2}x + 6$
43. $y = \frac{1}{2}x - 4$
44. $y = \frac{2}{3}x + 1$
45. $y = 9 - x^2$
46. $y = x^2 + 3$
47. $y = (x + 3)^2$
48. $y = 2x^2 + x$
49. $y = x^3 + 2$
50. $y = x^3 - 4x$
51. $y = x\sqrt{x + 5}$
52. $y = \sqrt{25 - x^2}$
53. $x = y^3$
54. $x = y^2 - 4$
55. $y = \frac{8}{x}$
56. $y = \frac{10}{x^2 + 1}$
57. $y = 6 - |x|$
58. $y = |6 - x|$

In Exercises 59–62, use a graphing utility to graph the equation. Identify any intercepts and test for symmetry.

59. $y^2 - x = 9$
60. $x^2 + 4y^2 = 4$
61. $x + 3y^2 = 6$
62. $3x - 4y^2 = 8$

In Exercises 63–70, find the points of intersection of the graphs of the equations.

63. $x + y = 8$
 $4x - y = 7$
64. $3x - 2y = -4$
 $4x + 2y = -10$
65. $x^2 + y = 6$
 $x + y = 4$
66. $x = 3 - y^2$
 $y = x - 1$

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

67. $x^2 + y^2 = 5$
$x - y = 1$

68. $x^2 + y^2 = 25$
$-3x + y = 15$

69. $y = x^3$
$y = x$

70. $y = x^3 - 4x$
$y = -(x + 2)$

In Exercises 71–74, use a graphing utility to find the points of intersection of the graphs. Check your results analytically.

71. $y = x^3 - 2x^2 + x - 1$
$y = -x^2 + 3x - 1$

72. $y = x^4 - 2x^2 + 1$
$y = 1 - x^2$

73. $y = \sqrt{x + 6}$
$y = \sqrt{-x^2 - 4x}$

74. $y = -|2x - 3| + 6$
$y = 6 - x$

75. *Modeling Data* The table shows the Consumer Price Index (CPI) for selected years. *(Source: Bureau of Labor Statistics)*

Year	1975	1980	1985	1990	1995	2000	2005
CPI	53.8	82.4	107.6	130.7	152.4	172.2	195.3

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at^2 + bt + c$ for the data. In the model, y represents the CPI and t represents the year, with $t = 5$ corresponding to 1975.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the CPI for the year 2010.

76. *Modeling Data* The table shows the numbers of cellular phone subscribers (in millions) in the United States for selected years. *(Source: Cellular Telecommunications and Internet Association)*

Year	1990	1993	1996	1999	2002	2005
Number	5	16	44	86	141	208

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at^2 + bt + c$ for the data. In the model, y represents the number of subscribers and t represents the year, with $t = 0$ corresponding to 1990.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the number of cellular phone subscribers in the United States in the year 2015.

77. *Break-Even Point* Find the sales necessary to break even ($R = C$) if the cost C of producing x units is

$C = 5.5\sqrt{x} + 10{,}000$ Cost equation

and the revenue R from selling x units is

$R = 3.29x.$ Revenue equation

78. *Copper Wire* The resistance y in ohms of 1000 feet of solid copper wire at 77°F can be approximated by the model

$$y = \frac{10{,}770}{x^2} - 0.37, \quad 5 \le x \le 100$$

where x is the diameter of the wire in mils (0.001 in.). Use a graphing utility to graph the model. If the diameter of the wire is doubled, the resistance is changed by about what factor?

WRITING ABOUT CONCEPTS

In Exercises 79 and 80, write an equation whose graph has the indicated property. (There may be more than one correct answer.)

79. The graph has intercepts at $x = -4$, $x = 3$, and $x = 8$.

80. The graph has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

81. (a) Prove that if a graph is symmetric with respect to the x-axis and to the y-axis, then it is symmetric with respect to the origin. Give an example to show that the converse is not true.

(b) Prove that if a graph is symmetric with respect to one axis and to the origin, then it is symmetric with respect to the other axis.

CAPSTONE

82. Match the equation or equations with the given characteristic.

(i) $y = 3x^3 - 3x$ (ii) $y = (x + 3)^2$ (iii) $y = 3x - 3$
(iv) $y = \sqrt[3]{x}$ (v) $y = 3x^2 + 3$ (vi) $y = \sqrt{x + 3}$

(a) Symmetric with respect to the y-axis
(b) Three x-intercepts
(c) Symmetric with respect to the x-axis
(d) $(-2, 1)$ is a point on the graph
(e) Symmetric with respect to the origin
(f) Graph passes through the origin

True or False? **In Exercises 83–86, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

83. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the x-axis, then $(4, -5)$ is also a point on the graph.

84. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the y-axis, then $(4, -5)$ is also a point on the graph.

85. If $b^2 - 4ac > 0$ and $a \ne 0$, then the graph of $y = ax^2 + bx + c$ has two x-intercepts.

86. If $b^2 - 4ac = 0$ and $a \ne 0$, then the graph of $y = ax^2 + bx + c$ has only one x-intercept.

In Exercises 87 and 88, find an equation of the graph that consists of all points (x, y) having the given distance from the origin. (For a review of the Distance Formula, see Appendix C.)

87. The distance from the origin is twice the distance from $(0, 3)$.

88. The distance from the origin is K $(K \ne 1)$ times the distance from $(2, 0)$.

P.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–6, estimate the slope of the line from its graph. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

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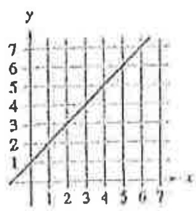

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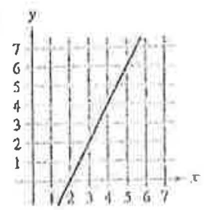

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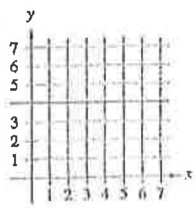

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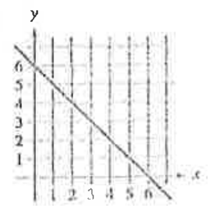

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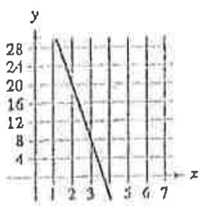

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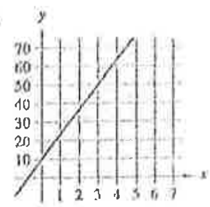


In Exercises 7 and 8, sketch the lines through the point with the indicated slopes. Make the sketches on the same set of coordinate axes.

	Point	*Slopes*			
7.	$(3, 4)$	(a) 1	(b) -2	(c) $-\frac{3}{2}$	(d) Undefined
8.	$(-2, 5)$	(a) 3	(b) -3	(c) $\frac{1}{3}$	(d) 0

In Exercises 9–14, plot the pair of points and find the slope of the line passing through them.

9. $(3, -4), (5, 2)$
10. $(1, 1), (-2, 7)$
11. $(4, 6), (4, 1)$
12. $(3, -5), (5, -5)$
13. $\left(-\frac{1}{2}, \frac{2}{3}\right), \left(-\frac{3}{4}, \frac{1}{6}\right)$
14. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

In Exercises 15–18, use the point on the line and the slope of the line to find three additional points that the line passes through. (There is more than one correct answer.)

	Point	*Slope*		*Point*	*Slope*
15.	$(6, 2)$	$m = 0$	16.	$(-4, 3)$	m is undefined.
17.	$(1, 7)$	$m = -3$	18.	$(-2, -2)$	$m = 2$

19. *Conveyor Design* A moving conveyor is built to rise 1 meter for each 3 meters of horizontal change.
    (a) Find the slope of the conveyor.
    (b) Suppose the conveyor runs between two floors in a factory. Find the length of the conveyor if the vertical distance between floors is 10 feet.

20. *Rate of Change* Each of the following is the slope of a line representing daily revenue y in terms of time x in days. Use the slope to interpret any change in daily revenue for a one-day increase in time.
    (a) $m = 800$ (b) $m = 250$ (c) $m = 0$

21. *Modeling Data* The table shows the populations y (in millions) of the United States for 2000 through 2005. The variable t represents the time in years, with $t = 0$ corresponding to 2000. *(Source: U.S. Bureau of the Census)*

t	0	1	2	3	4	5
y	282.4	285.3	288.2	291.1	293.9	296.6

    (a) Plot the data by hand and connect adjacent points with a line segment.
    (b) Use the slope of each line segment to determine the year when the population increased least rapidly.

22. *Modeling Data* The table shows the rate r (in miles per hour) that a vehicle is traveling after t seconds.

t	5	10	15	20	25	30
r	57	74	85	84	61	43

    (a) Plot the data by hand and connect adjacent points with a line segment.
    (b) Use the slope of each line segment to determine the interval when the vehicle's rate changed most rapidly. How did the rate change?

In Exercises 23–28, find the slope and the y-intercept (if possible) of the line.

23. $y = 4x - 3$
24. $-x + y = 1$
25. $x + 5y = 20$
26. $6x - 5y = 15$
27. $x = 4$
28. $y = -1$

In Exercises 29–34, find an equation of the line that passes through the point and has the indicated slope. Sketch the line.

	Point	*Slope*		*Point*	*Slope*
29.	$(0, 3)$	$m = \frac{3}{4}$	30.	$(-5, -2)$	m is undefined.
31.	$(0, 0)$	$m = \frac{2}{3}$	32.	$(0, 4)$	$m = 0$
33.	$(3, -2)$	$m = 3$	34.	$(-2, 4)$	$m = -\frac{3}{5}$

In Exercises 35–44, find an equation of the line that passes through the points, and sketch the line.

35. $(0, 0), (4, 8)$
36. $(0, 0), (-1, 5)$
37. $(2, 1), (0, -3)$
38. $(-2, -2), (1, 7)$
39. $(2, 8), (5, 0)$
40. $(-3, 6), (1, 2)$
41. $(6, 3), (6, 8)$
42. $(1, -2), (3, -2)$
43. $\left(\frac{1}{2}, \frac{7}{2}\right), \left(0, \frac{3}{4}\right)$
44. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

45. Find an equation of the vertical line with x-intercept at 3.

46. Show that the line with intercepts $(a, 0)$ and $(0, b)$ has the following equation.

$$\frac{x}{a} + \frac{y}{b} = 1, \quad a \neq 0, b \neq 0$$

In Exercises 47–50, use the result of Exercise 46 to write an equation of the line in general form.

47. x-intercept: $(2, 0)$
    y-intercept: $(0, 3)$
48. x-intercept: $\left(-\frac{2}{3}, 0\right)$
    y-intercept: $(0, -2)$
49. Point on line: $(1, 2)$
    x-intercept: $(a, 0)$
    y-intercept: $(0, a)$
    $(a \neq 0)$
50. Point on line: $(-3, 4)$
    x-intercept: $(a, 0)$
    y-intercept: $(0, a)$
    $(a \neq 0)$

In Exercises 51–58, sketch a graph of the equation.

51. $y = -3$
52. $x = 4$
53. $y = -2x + 1$
54. $y = \frac{1}{3}x - 1$
55. $y - 2 = \frac{3}{2}(x - 1)$
56. $y - 1 = 3(x + 4)$
57. $2x - y - 3 = 0$
58. $x + 2y + 6 = 0$

59. *Square Setting* Use a graphing utility to graph the lines $y = 2x - 3$ and $y = -\frac{1}{2}x + 1$ in each viewing window. Compare the graphs. Do the lines appear perpendicular? Are the lines perpendicular? Explain.

(a)
Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -5
Ymax = 5
Yscl = 1

(b)
Xmin = -6
Xmax = 6
Xscl = 1
Ymin = -4
Ymax = 4
Yscl = 1

CAPSTONE

60. A line is represented by the equation $ax + by = 4$.
    (a) When is the line parallel to the x-axis?
    (b) When is the line parallel to the y-axis?
    (c) Give values for a and b such that the line has a slope of $\frac{5}{8}$.
    (d) Give values for a and b such that the line is perpendicular to $y = \frac{2}{5}x + 3$.
    (e) Give values for a and b such that the line coincides with the graph of $5x + 6y = 8$.

In Exercises 61–66, write the general forms of the equations of the lines through the point (a) parallel to the given line and (b) perpendicular to the given line.

	Point	*Line*		*Point*	*Line*
61.	$(-7, -2)$	$x = 1$	62.	$(-1, 0)$	$y = -3$
63.	$(2, 1)$	$4x - 2y = 3$	64.	$(-3, 2)$	$x + y = 7$
65.	$\left(\frac{3}{4}, \frac{7}{8}\right)$	$5x - 3y = 0$	66.	$(4, -5)$	$3x + 4y = 7$

Rate of Change In Exercises 67–70, you are given the dollar value of a product in 2008 *and* the rate at which the value of the product is expected to change during the next 5 years. Write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 0$ represent 2000.)

	2008 Value	*Rate*
67.	\$1850	\$250 increase per year
68.	\$156	\$4.50 increase per year
69.	\$17,200	\$1600 decrease per year
70.	\$245,000	\$5600 decrease per year

In Exercises 71 and 72, use a graphing utility to graph the parabolas and find their points of intersection. Find an equation of the line through the points of intersection and graph the line in the same viewing window.

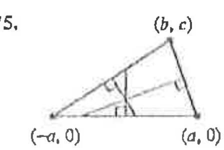
71. $y = x^2$
    $y = 4x - x^2$
72. $y = x^2 - 4x + 3$
    $y = -x^2 + 2x + 3$

In Exercises 73 and 74, determine whether the points are collinear. (Three points are *collinear* if they lie on the same line.)

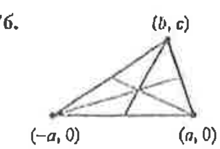
73. $(-2, 1), (-1, 0), (2, -2)$
74. $(0, 4), (7, -6), (-5, 11)$

WRITING ABOUT CONCEPTS

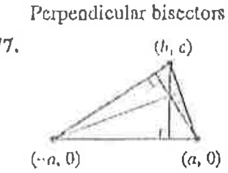
In Exercises 75–77, find the coordinates of the point of intersection of the given segments. Explain your reasoning.

75.


Perpendicular bisectors

76.


Medians

77.


Altitudes

78. Show that the points of intersection in Exercises 75, 76, and 77 are collinear.

P.3 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1 and 2, use the graphs of f and g to answer the following.

(a) Identify the domains and ranges of f and g.

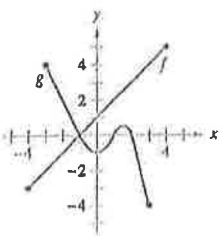
(b) Identify $f(-2)$ and $g(3)$.

(c) For what value(s) of x is $f(x) = g(x)$?

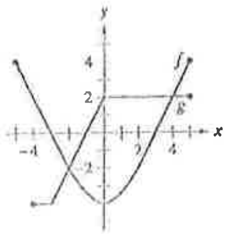
(d) Estimate the solution(s) of $f(x) = 2$.

(e) Estimate the solutions of $g(x) = 0$.

1.



2.



In Exercises 3–12, evaluate (if possible) the function at the given value(s) of the independent variable. Simplify the results.

3. $f(x) = 7x - 4$
   (a) $f(0)$
   (b) $f(-3)$
   (c) $f(b)$
   (d) $f(x - 1)$

4. $f(x) = \sqrt{x + 5}$
   (a) $f(-4)$
   (b) $f(11)$
   (c) $f(-8)$
   (d) $f(x + \Delta x)$

5. $g(x) = 5 - x^2$
   (a) $g(0)$
   (b) $g(\sqrt{5})$
   (c) $g(-2)$
   (d) $g(t - 1)$

6. $g(x) = x^2(x - 4)$
   (a) $g(4)$
   (b) $g(\frac{3}{2})$
   (c) $g(c)$
   (d) $g(t + 4)$

7. $f(x) = \cos 2x$
   (a) $f(0)$
   (b) $f(-\pi/4)$
   (c) $f(\pi/3)$

8. $f(x) = \sin x$
   (a) $f(\pi)$
   (b) $f(5\pi/4)$
   (c) $f(2\pi/3)$

9. $f(x) = x^3$
   $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

10. $f(x) = 3x - 1$
   $\dfrac{f(x) - f(1)}{x - 1}$

11. $f(x) = \dfrac{1}{\sqrt{x - 1}}$
   $\dfrac{f(x) - f(2)}{x - 2}$

12. $f(x) = x^3 - x$
   $\dfrac{f(x) - f(1)}{x - 1}$

In Exercises 13–20, find the domain and range of the function.

13. $f(x) = 4x^2$
14. $g(x) = x^2 - 5$
15. $g(x) = \sqrt{6x}$
16. $h(x) = -\sqrt{x + 3}$
17. $f(t) = \sec \dfrac{\pi t}{4}$
18. $h(t) = \cot t$
19. $f(x) = \dfrac{3}{x}$
20. $g(x) = \dfrac{2}{x - 1}$

In Exercises 21–26, find the domain of the function.

21. $f(x) = \sqrt{x} + \sqrt{1 - x}$
22. $f(x) = \sqrt{x^2 - 3x + 2}$
23. $g(x) = \dfrac{2}{1 - \cos x}$
24. $h(x) = \dfrac{1}{\sin x - \frac{1}{2}}$
25. $f(x) = \dfrac{1}{|x + 3|}$
26. $g(x) = \dfrac{1}{|x^2 - 4|}$

In Exercises 27–30, evaluate the function as indicated. Determine its domain and range.

27. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$
   (a) $f(-1)$ (b) $f(0)$ (c) $f(2)$ (d) $f(t^2 + 1)$

28. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$
   (a) $f(-2)$ (b) $f(0)$ (c) $f(1)$ (d) $f(s^2 + 2)$

29. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \ge 1 \end{cases}$
   (a) $f(-3)$ (b) $f(1)$ (c) $f(3)$ (d) $f(b^2 + 1)$

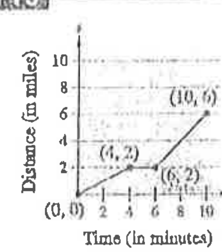
30. $f(x) = \begin{cases} \sqrt{x + 4}, & x \le 5 \\ (x - 5)^2, & x > 5 \end{cases}$
   (a) $f(-3)$ (b) $f(0)$ (c) $f(5)$ (d) $f(10)$

In Exercises 31–38, sketch a graph of the function and find its domain and range. Use a graphing utility to verify your graph.

31. $f(x) = 4 - x$
32. $g(x) = \dfrac{4}{x}$
33. $h(x) = \sqrt{x - 6}$
34. $f(x) = \frac{1}{4}x^3 + 3$
35. $f(x) = \sqrt{9 - x^2}$
36. $f(x) = x + \sqrt{4 - x^2}$
37. $g(t) = 3 \sin \pi t$
38. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

WRITING ABOUT CONCEPTS

39. The graph of the distance that a student drives in a 10-minute trip to school is shown in the figure. Give a verbal description of characteristics of the student's drive to school.

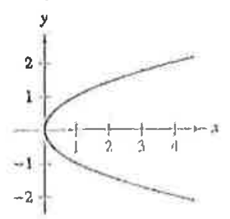


WRITING ABOUT CONCEPTS (continued)

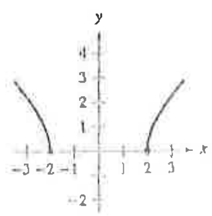
40. A student who commutes 27 miles to attend college remembers, after driving a few minutes, that a term paper that is due has been forgotten. Driving faster than usual, the student returns home, picks up the paper, and once again starts toward school. Sketch a possible graph of the student's distance from home as a function of time.

In Exercises 41–44, use the Vertical Line Test to determine whether y is a function of x. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

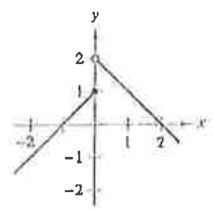
41. $x - y^2 = 0$



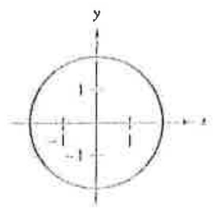
42. $\sqrt{x^2 - 4} - y = 0$



43. $y = \begin{cases} x + 1, & x \le 0 \\ -x + 2, & x > 0 \end{cases}$



44. $x^2 + y^2 = 4$



In Exercises 45–48, determine whether y is a function of x.

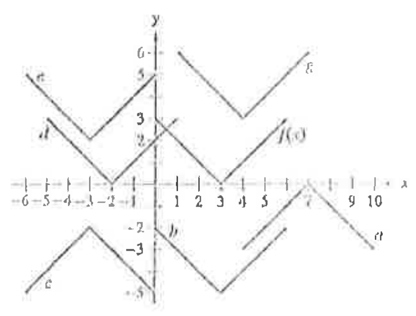
45. $x^2 + y^2 = 16$

46. $x^2 + y = 16$

47. $y^2 = x^2 - 1$

48. $x^2y - x^2 + 4y = 0$

In Exercises 49–54, use the graph of $y = f(x)$ to match the function with its graph.



49. $y = f(x + 5)$

50. $y = f(x) - 5$

51. $y = -f(-x) - 2$

52. $y = -f(x - 4)$

53. $y = f(x + 6) + 2$

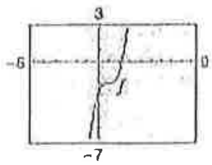
54. $y = f(x - 1) + 3$

55. Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x + 3)$ (b) $f(x - 1)$

(c) $f(x) + 2$ (d) $f(x) - 4$

(e) $3f(x)$ (f) $\frac{1}{4}f(x)$

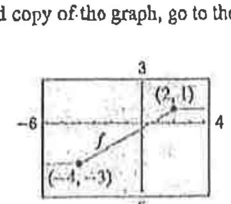


56. Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x - 4)$ (b) $f(x + 2)$

(c) $f(x) + 4$ (d) $f(x) - 1$

(e) $2f(x)$ (f) $\frac{1}{2}f(x)$



57. Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of each function. In each case, describe the transformation.

(a) $y = \sqrt{x} + 2$ (b) $y = -\sqrt{x}$ (c) $y = \sqrt{x - 2}$

58. Specify a sequence of transformations that will yield each graph of h from the graph of the function $f(x) = \sin x$.

(a) $h(x) = \sin\left(x + \dfrac{\pi}{2}\right) + 1$ (b) $h(x) = -\sin(x - 1)$

59. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, evaluate each expression.

(a) $f(g(1))$ (b) $g(f(1))$ (c) $g(f(0))$

(d) $f(g(-4))$ (e) $f(g(x))$ (f) $g(f(x))$

60. Given $f(x) = \sin x$ and $g(x) = \pi x$, evaluate each expression.

(a) $f(g(2))$ (b) $f\left(g\left(\dfrac{1}{2}\right)\right)$ (c) $g(f(0))$

(d) $g\left(f\left(\dfrac{\pi}{4}\right)\right)$ (e) $f(g(x))$ (f) $g(f(x))$

In Exercises 61–64, find the composite functions $(f \circ g)$ and $(g \circ f)$. What is the domain of each composite function? Are the two composite functions equal?

61. $f(x) = x^2$
$g(x) = \sqrt{x}$

62. $f(x) = x^2 - 1$
$g(x) = \cos x$

63. $f(x) = \dfrac{3}{x}$
$g(x) = x^2 - 1$

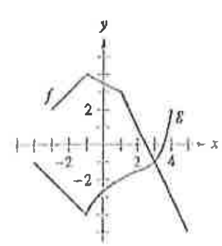
64. $f(x) = \dfrac{1}{x}$
$g(x) = \sqrt{x + 2}$

65. Use the graphs of f and g to evaluate each expression. If the result is undefined, explain why.

(a) $(f \circ g)(3)$ (b) $g(f(2))$

(c) $g(f(5))$ (d) $(f \circ g)(-3)$

(e) $(g \circ f)(-1)$ (f) $f(g(-1))$



DATE ______________________

1.2–1.6 Concepts Worksheet

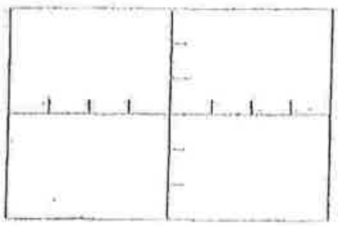
NAME ______________________

Graphical Analysis

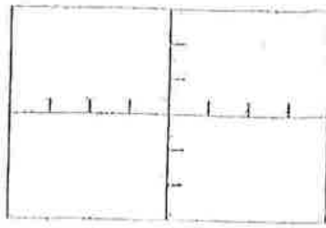
Chapter 1 deals with functions and their graphical characteristics. To facilitate a study of functions, it is important to visualize mentally the graph of a function when given an algebraic description.

1. Graph each function. Clearly indicate units on the axes provided.

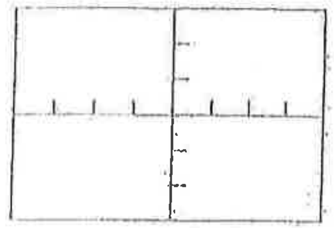
(a) $f(x) = x^2$



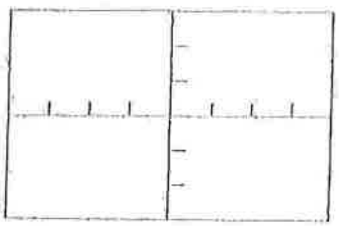
(b) $f(x) = x^3$



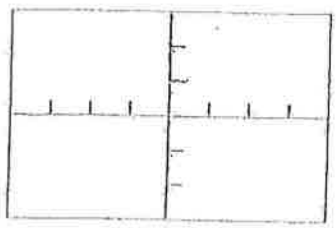
(c) $f(x) = |x|$



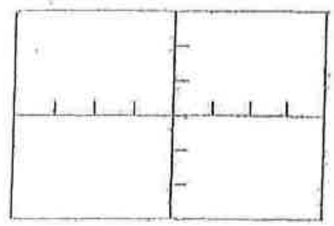
(d) $f(x) = \sin x$



(e) $f(x) = \cos x$

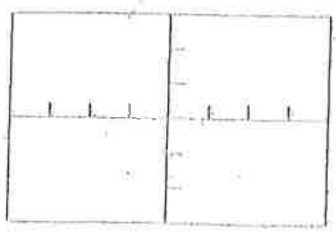


(f) $f(x) = \tan x$

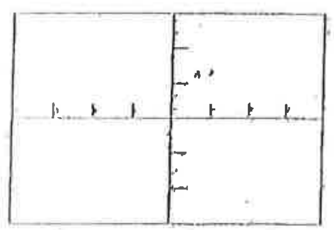


(g) $f(x) = \sec x$

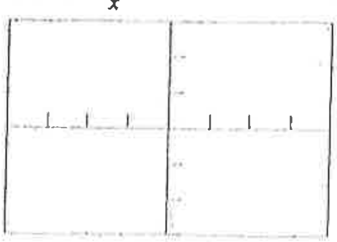
(h) $f(x) = 2^x$



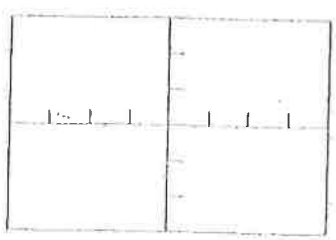
(i) $f(x) = \log_2 x$



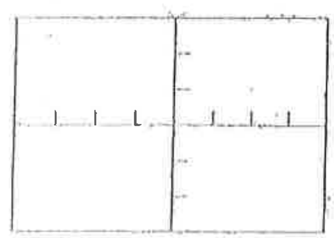
(j) $f(x) = \dfrac{1}{x}$



(k) $f(x) = \sqrt{x}$



(l) $f(x) = \sqrt{a^2 - x^2}$



Concepts Worksheet NAME

Continued

2. Answer the following questions about the indicated functions. In completing the table below, you may use the following abbreviations, *R*: the set of real numbers, *J*: the set of integers, and *N*: the set of natural numbers. Note: This exercise may need to be done as appropriate sections of Chapter 1 are completed.

Function	Domain	Range $y = f(x)$	Zeros (Find x when $f(x) = 0$)	Symmetry with respect to y-axis or origin	Even or Odd Function— $f(-x) = f(x)$ or $f(-x) = -f(x)$	Is the function periodic? If so, state the period.	Is $f(x)$ a one-to-one function? (For each $f(x)$ only one x exists)
(a) $f(x) = x^2$							
(b) $f(x) = x^3$							
(c) $f(x) = \|x\|$							
(d) $f(x) = \sin x$							
(e) $f(x) = \cos x$							
(f) $f(x) = \tan x$							
(g) $f(x) = \sec x$							
(h) $f(x) = 2^x$							
(i) $f(x) = \log_2 x$							
(j) $f(x) = \frac{1}{x}$							
(k) $f(x) = \sqrt{x}$							
(l) $f(x) = \sqrt{a^2 - x^2}$							

1.2-1.6 Concepts Worksheet NAME

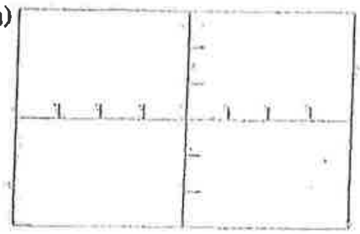
Continued

Concept Connectors

3. Is there a relationship between symmetry in a function's graph and the function's being even or odd? Explain.

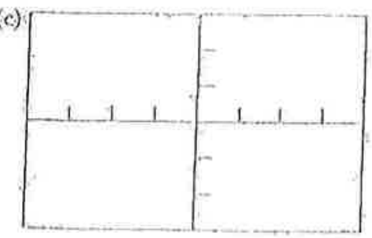
4. Draw a reflection of (a) $f(x) = \sin x$, (b) $f(x) = 2^x$ and (c) $f(x) = \sqrt{x}$ through the x-axis.

(a)



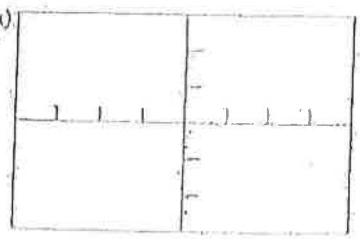
(b)

(c)

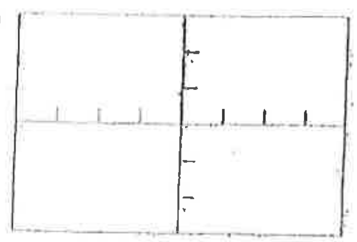


5. Draw a reflection of (a) $f(x) = \sin x$, (b) $f(x) = 2^x$ and (c) $f(x) = \sqrt{x}$ through the y-axis.

(a)



(b)



(c)